

AFCRC-TN-59-144

ASTIA DOCUMENT No. AD 211808

NEW YORK UNIVERSITY
INSTITUTE OF MATHEMATICAL SCIENCES
LIBRARY

25 Waverly Place, New York 3, N. Y.



NEW YORK UNIVERSITY

Institute of Mathematical Sciences

Division of Electromagnetic Research

NOV 18 1959

RESEARCH REPORT No. EM-127

Back Scattering From a Finite Cone

JOSEPH B. KELLER

Contract No. AF19(604)1717

FEBRUARY, 1959

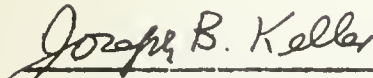
NEW YORK UNIVERSITY

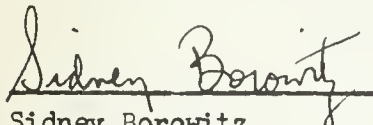
Institute of Mathematical Sciences
Division of Electromagnetic Research

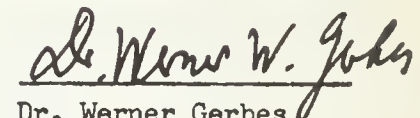
RESEARCH REPORT No. EM-127

BACK SCATTERING FROM A FINITE CONE

Joseph B. Keller


Joseph B. Keller


Sidney Borowitz
Acting Director


Dr. Werner Gerbes
Contract Monitor

The research reported in this document has been sponsored by the Electronics Research Directorate of the Air Force Cambridge Research Center, Air Research and Development Command. The publication of this report does not necessarily constitute approval by the Air Force of the findings or conclusions contained herein.

Contract No. AF 19(604)1717

February, 1959

Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies should be directed to the:

ARMED SERVICES TECHNICAL INFORMATION AGENCY
DOCUMENTS SERVICE CENTER
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

Department of Defense contractors must be established for ASTIA services or have their 'need-to-know' certified by the cognizant military agency of their project or contract.

All other persons and organizations should apply to the:

U.S. DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES
WASHINGTON 25, D.C.

Abstract*

Back scattering is calculated for an acoustic wave incident on a hard or soft finite cone, and an electromagnetic wave incident on a perfectly conducting finite cone. Two shapes of cone are treated, one with a flat base and the other with a rounded base. The calculation is based on the geometrical theory of diffraction. Therefore it is valid for wavelengths as large as the cone dimensions or smaller. Graphs of the back scattering cross-section versus cone angle and versus wavelength are given for axial incidence on the flat-based cone. Suggestions for shaping an object to minimize its back scattering are also included.

Table of Contents

1. Introduction	1
2. Qualitative description	1a
3. Flat based cone - single diffraction	3
4. Flat based cone - double diffraction	10
5. Cone with a spherical base	13
6. Conclusion	15
References	17

*Some of these results were contained in a report entitled "Radar return from metallic objects" written in January, 1958, when the author was a consultant to the General Electric Co., Missiles and Ordnance Systems Dept. Aerodynamics Lab., Communications Section. Most of the other results were contained in a paper with the present title read at the Washington meeting of U.R.S.I. in May, 1958.

1. Introduction

The back scattering from a finite cone is calculated in this report by means of the geometrical theory of diffraction^{[1], [2]} Three different physical problems are considered. One is that of an acoustic wave incident upon a rigid cone. The second is that of an acoustic wave incident upon a perfectly soft (pressure-release) cone. The third, and perhaps most important, is that of an electromagnetic wave incident upon a perfectly conducting cone. In all cases two different types of cones are treated. One is a truncated right circular cone with a flat base. The other has a rounded base which is a portion of a sphere. The sphere and cone join smoothly, with no sharp edge. For simplicity the incident wave is assumed to be plane and only the back-scattered field is found, although both of these limitations could easily be removed.

The theory employed here represents the back-scattered field as a superposition of fields on various rays. Some of these are the ordinary optical rays which are reflected from the cone and the others are diffracted rays. In the next section the various rays are described and a qualitative description of the back scattering process is given. In Sections 3-5 the quantitative results are derived and examined. Graphs of the back scattering cross section are also presented. In the conclusion, Section 6, suggestions are made about shaping an object to minimize its back-scattering cross section.

Most of the previous theoretical results on back scattering from cones have concerned semi-infinite cones. For such cones the acoustic diffraction problem was solved exactly and explicitly by H. S. Carslaw^[3] and the electromagnetic problem by W. W. Hansen and L. I. Schiff^[4]. Their solutions were given as infinite series which were difficult to evaluate. For the case of axial incidence, the back scattered fields

were evaluated for small angle and large angle cones by C. E. Schensted^[5], K. M. Siegel, J.W. Crispin and C. E. Schensted^[6] and L. B. Felsen^[7],^[8]. For off axial incidence on small angle cones, they were evaluated by L. B. Felsen^[9]. For axial incidence, Felsen^[10] has also evaluated the back scattered fields approximately for cones of any angle. Another approximate result for the back scattered field was found by R. C. Spenser^[11] using the Kirchhoff method. All of these results pertain to the field diffracted from the tip and yield what we call the tip diffraction coefficient. As we shall see, this field is not important for the flat-based cone, but it is important for the round-based cone.

The kirchhoff method has been applied to the finite cone by K. M. Siegel, et al^[12]. Siegel^[13] has also treated the case of axial incidence on the flat-based finite cone by a variant of our method. However, his result is incorrect because of an error in his formula for the field scattered from a wedge.

2. Qualitative description

Let us consider a plane wave incident upon a finite cone with a flat base. We denote the half-angle of the cone by γ and the radius of the base by a . (See Figure 1.) Let ϕ denote the angle between the incident rays and the axis of the cone, with the range of ϕ given by $0 \leq \phi \leq \pi$. The extreme values $\phi = 0$ and $\phi = \pi$ denote incidence along the axis, upon

the tip and base respectively. When $\phi = \pi$ the incident rays hit the base normally and therefore produce specularly reflected rays which return to the source. When $\phi = \frac{\pi}{2} - \gamma$ the incident rays are normal to the curved surface of the cone and also give rise to specularly reflected rays which return to the source. For all other values of ϕ no specularly reflected rays are returned to the source. As a consequence the back scattering is strongest at $\phi = \pi$ and $\phi = \frac{\pi}{2} - \gamma$.

When $\phi \neq \pi, \frac{\pi}{2} - \gamma$ the only back scattered rays are diffracted ones. The field on a ray diffracted from the tip is proportional to k^{-1} , that on a ray diffracted from the edge is proportional to $k^{-1/2}$, and that on a surface diffracted ray decreases exponentially with k . Therefore for large k , or more precisely for large ka , the edge diffracted rays are most important, the tip diffracted rays are next and the surface rays least. Rays doubly diffracted at edges also have fields proportional to k^{-1} so they are equal in importance to tip diffracted rays. When $\phi = 0$ or π the edge diffracted rays have a caustic on the axis, so their field there is multiplied by $k^{1/2}$. In this case the singly diffracted edge rays contribute a field proportional to k^0 and the doubly diffracted ones a field proportional to $k^{-1/2}$. Thus when $\phi = 0$ the doubly diffracted edge rays are more important than the singly diffracted tip ray. In addition the amplitude of the singly diffracted edge field is independent of k , just like the incident field.

When ϕ lies in the interval $0 < \phi < \gamma$ or the interval $\frac{\pi}{2} < \phi < \pi$, two rays singly diffracted at the edge are returned to the source. Therefore in these ranges the back scattered field will be an oscillatory function of ka , due to interference between these two rays. When ϕ lies in the interval $\gamma < \phi < \frac{\pi}{2}$ only one ray singly diffracted at the edge returns to

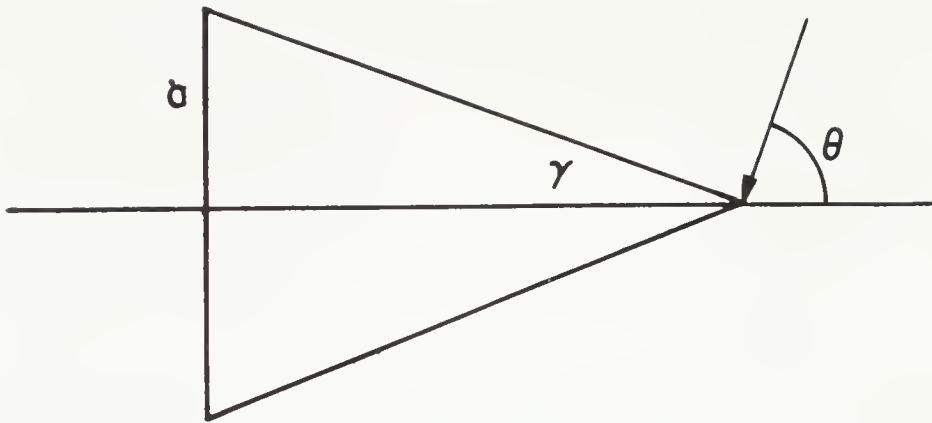


Figure 1.

Cross section of a finite cone of half-angle γ with a flat base of radius a . The tip of the cone is the origin of a polar coordinate system and the angle ϕ is measured from the axis.

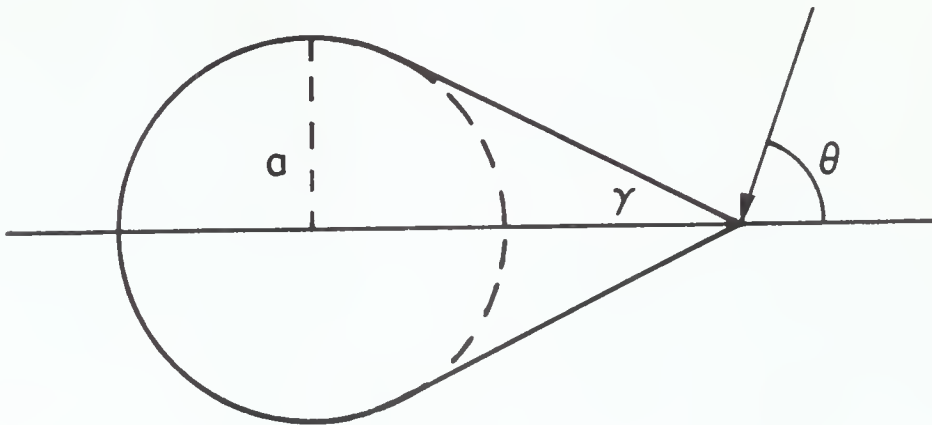


Figure 2.

Cross section of a finite cone of half-angle γ with a spherical base of radius a . The cone is tangential to the sphere where they join.

the source. Therefore the field will not oscillate as ka varies, except for a small oscillation due to interference with a doubly diffracted ray. In all these cases, the amplitude of the field will be proportional to $(ka)^{-1/2}$.

Let us now consider the same problem for the finite cone of Figure 2. This cone has a spherical rear surface which joins the cone smoothly. Now a will denote the radius of the rear surface. In this case specularly reflected rays are returned to the source for all values of ϕ in the range $\frac{\pi}{2} - \gamma \leq \phi \leq \pi$. In the range $0 \leq \phi < \frac{\pi}{2} - \gamma$ only tip and surface diffracted rays return to the source. When $\phi = 0$ the surface diffracted rays have a caustic on the axis so their field there is multiplied by $k^{1/2}$. The relative importance of these two kinds of rays depends upon the value of ka . For very large values of ka , the tip ray is more important while for smaller values of ka the surface rays are dominant. When the tip ray is dominant, interference does not occur, so the field does not oscillate as ka varies.

3. Flat based cone - single diffraction

We shall first determine the back scattered field for the flat-based cone when $\phi \neq 0, \frac{\pi}{2} - \gamma, \pi$. The field on rays singly diffracted from the edge will be called u_{e1} . According to reference [2], it is given by

$$(1) \quad u_{e1}(P) = \left[r_1(1+\rho_1^{-1}r_1) \right]^{-1/2} e^{ikr_1} D_1 u_1(P_1) + \left[r_2(1+\rho_2^{-1}r_2) \right]^{-1/2} e^{ikr_2} D_2 u_1(P_2).$$

In (1), P denotes the point at which the field is being determined while P_1 and P_2 denote the two points on the edge from which rays are diffracted to P . The point on the edge nearest to P is P_1 and its distance from P is r_1 .

D_1 is the diffraction coefficient which depends upon the directions of the incident and diffracted rays at P_1 . The non-vanishing radius of curvature of the diffracted wavefront at P_1 is ρ_1 , and the incident field is u_i . The corresponding quantities with the subscript '2' are evaluated at P_2 , the point on the edge farthest from P. When ϕ lies in the range $\gamma < \phi < \frac{\pi}{2}$ only the P_1 term is present in (1) since then the point P_2 is hidden by the cone.

To evaluate the right side of (1) let us introduce the polar coordinates r, θ . In terms of them the incident field is assumed to be given by

$$(2) \quad u_i(r, \theta) = A e^{-ikr \cos(\theta - \phi)}.$$

In (2) A is a constant, which is a vector in the electromagnetic case.

The coordinates of P_1 are a csc γ and $\pi - \gamma$ so $u_i(P_1) = A \exp \left[ikacsc\gamma \cos(\gamma + \phi) \right]$. Similarly $u_i(P_2) = A \exp \left[ikacsc\gamma \cos(\gamma - \phi) \right]$. The radius ρ_1 is just the signed distance along the diffracted ray from P_1 to the axis, which is a caustic of the diffracted rays. A simple geometrical calculation yields

$$(3) \quad (1 + \rho_1^{-1} r_1) = \frac{r \sin \theta}{a}$$

$$(4) \quad (1 + \rho_2^{-1} r_2) = \frac{-r \sin \theta}{a}.$$

To calculate D_1 for an acoustic wave we utilize (A10) of reference [2]*. In that equation we must set $\beta = \pi/2$ because the incident ray is normally incident on the edge. We also set $\theta = \alpha$ because we will consider only back scattering. We find at once that at P_1 , $\alpha = \pi - \phi$ while at P_2 , $\alpha = \pi + \phi$. Thus we obtain

*Equation (A10) of reference [2] is written incorrectly and should read

$$D = \frac{e^{i\pi/4} \sin(\pi/n)}{n \sin \beta \sqrt{2\pi k}} \left[\left(\cos \frac{\pi}{n} - \cos \frac{\theta - \alpha}{n} \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{\theta + \alpha + \pi}{n} \right)^{-1} \right].$$

$$(5) \quad D_1 = \frac{e^{in/4} \sin(\pi/n)}{n \sqrt{2\pi k}} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi-2\phi}{n} \right)^{-1} \right].$$

Here $n = \frac{3}{2} + \frac{\gamma}{\pi}$. The upper sign in (5) applies to the cone on which the field vanishes (the soft cone) while the lower sign applies to one on which the normal derivative vanishes (the hard cone). To obtain D_2 we need merely change the sign of ϕ in (5). In the electromagnetic case these same diffraction coefficients can be used, the upper sign applying to the electric field component parallel to the edge and the lower sign to the component normal to the edge.

For large r it is convenient to express r_1 and r_2 in the phase factors in terms of r and θ by $r_1 \approx r + a \csc \gamma \cos(\gamma+\phi)$ and $r_2 \approx r + a \csc \gamma \cos(\gamma-\phi)$. In the amplitudes it suffices to use $r_1 \approx r_2 \approx r$. Upon combining all these results, and setting $\theta = \phi$, we obtain from (1) in the scalar case

$$(6) \quad u_{el}(r, \phi) = \frac{\sqrt{a} \sin(\pi/n) A e^{ikr+2kacsc\gamma\cos(\gamma+\phi)+i\frac{\pi}{4}}}{nr \sqrt{2\pi k \sin \phi}} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi-2\phi}{n} \right)^{-1} \right].$$

This result holds for $\gamma < \phi < \frac{\pi}{2}$ when only one edge diffracted ray returns to the source. In the range $0 < \phi < \gamma$ and $\frac{\pi}{2} < \phi < \pi$ when two such rays occur, u_{el} is given by the right side of (6) plus the same expression with ϕ replaced by $-\phi$. Thus in this range

$$(7) \quad u_{el}(r, \phi) = \frac{\sqrt{a} \sin(\pi/n) A e^{ikr}}{nr \sqrt{2\pi k \sin \phi}} \left\{ e^{i \left[2kacsc\gamma\cos(\gamma+\phi) + \frac{\pi}{4} \right]} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi-2\phi}{n} \right)^{-1} \right] \right. \\ \left. + e^{i \left[2kacsc\gamma\cos(\gamma-\phi) - \frac{\pi}{4} \right]} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi+2\phi}{n} \right)^{-1} \right] \right\}.$$

(In the last term $3\pi+2\phi$ must be replaced by $2\phi-\pi$ when $\frac{\pi}{2} \leq \phi \leq \pi$.) In (6) and (7) the upper signs hold for the soft cone and the lower signs for the hard cone.

In the electromagnetic case (6) and (7) also apply. The soft cone result applies to the electric field component parallel to the edge at P_1 - i.e., perpendicular to the plane containing P and the axis. The hard cone result applies to the electric field component normal to the edge at P_1 - i.e., lying in the plane containing P and the axis.

The result (6) is not valid at $\phi = \frac{\pi}{2} - \gamma$ where it becomes infinite. For this value of ϕ the specularly reflected ray must also be taken into account. The result (7) is not valid at $\phi = 0, \pi$ where it becomes infinite. For these values of ϕ the diffracted rays form a caustic on the axis, and in addition at $\phi = \pi$ the specularly reflected rays must be considered.

Let us first obtain a modified form of (7) valid at $\phi = 0$. To this end we define $\rho = r \sin \theta$, which is the distance of P from the axis. We also define $\delta = \cos^{-1}(a/r)$ and note that $\rho \cos \delta = a \sin \theta$. Now we must reexamine (7) before setting θ equal to ϕ , with $\phi = 0$. Therefore we return to (1), set $u_1(P_1) = u_1(P_2)$ and $D_1 = D_2$, use (3) and (4), and in the phase set $r_1 = (r \cos \theta + a \cot \gamma) \sin \delta + (a - \rho) \cos \delta$. For r_2 we use the same formula with $-\rho$ in place of ρ . Then (1) becomes, after some simplification, instead of (7)

$$(8) \quad u_{el}(r, \theta) = \frac{a \sin(\pi/n) A e^{2ikacot\gamma}}{nr \sqrt{2\pi k \rho \cos \delta}} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{n} \right)^{-1} \right] \\ \left[e^{-ik(r \cos \theta \sin \delta - \rho \cos \delta) + i\frac{\pi}{4}} + e^{-ik(r \cos \theta \sin \delta + \rho \cos \delta) - i\frac{\pi}{4}} \right].$$

This is of the form given in (A15) of reference [2]. Therefore we may

apply the axial caustic correction given there. Upon doing so and eliminating δ , we obtain for θ small

$$(9) \quad u_{el}(r, \theta) = \frac{a \sin(\pi/n) A e^{ik(r+2a \cot \gamma)}}{nr} J_0(k a \sin \theta) \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{n} \right)^{-1} \right].$$

At $\theta = 0$, the Bessel function J_0 is equal to one. Thus at $\theta = 0$ (9) becomes the following, which is the back scattered field at $\phi = 0$

$$(10) \quad u_{el}(r, 0) = \frac{a \sin(\pi/n) A e^{ik(r+2a \cot \gamma)}}{nr} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{n} \right)^{-1} \right].$$

Now we consider $\phi = \pi$. Again we must return to (1) and keep $\theta \neq \pi$.

Proceeding as above we obtain from (1)

$$(11) \quad u_{el}(r, \theta) = \frac{a \sin(\pi/n) A e^{-2ik a \cot \gamma}}{nr \sqrt{2\pi k \rho \cos \delta}} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \left(\frac{\pi}{2n} + \frac{\delta}{n} \right) \right)^{-1} \right] \\ \left[e^{-ik(-r \cos \theta \sin \delta + [a-\rho] \cos \delta) + i \frac{\pi}{4}} + e^{ik(-r \cos \theta \sin \delta + [a+\rho] \cos \delta) - i \frac{\pi}{4}} \right].$$

Upon applying the caustic correction to (11), we obtain

$$(12) \quad u_{el}(r, \theta) = \frac{a \sin(\pi/n) A e^{-ik(r \cos \theta \sin \delta + 2a \cot \gamma)}}{nr} J_0(k a \sin \theta) \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} \right. \\ \left. + \left(\cos \frac{\pi}{n} - \cos \left(\frac{\pi}{2n} + \frac{\delta}{n} \right) \right)^{-1} \right].$$

Although (12) is finite at any finite distance r along the axis, it becomes infinite as r becomes infinite because then $\delta = \cos^{-1}(a/r)$ tends to $\pi/2$. Of course there is also a reflected field u_R on and near the axis, given by

$$(13) \quad u_R(r, \theta) = \mp A e^{-ik(r \cos \theta + 2a \cot \gamma)}, \quad |r \sin \theta| < a, \quad \theta > \frac{\pi}{2}.$$

The total field (u, r, θ) is the sum of the reflected and diffracted fields. Its form on the axis for large r can be computed from (12) and (13) by first setting $\theta = \pi$ and $\delta = (\pi/2) - (a/r)$. Then for large r ,

$$(14) \quad u = u_R + u_{e1} = A e^{ik(r-2a \cot \gamma)} \left[\mp 1 \mp \frac{a \sin(\pi/n)}{nr} \left(1 - \frac{ika^2}{2r} \right) \left(\cos \frac{\pi}{n} - \cos \left(\frac{\pi}{n} - \frac{a}{nr} \right) \right)^{-1} \right]$$

$$= \mp \frac{ika^2 A}{2r} e^{ik(r - 2a \cot \gamma)}.$$

Equations (6), (7), (10) and (14) give the back-scattered field for all directions of incidence except $\phi = \pi/2 - \gamma$. From these equations we can compute the back scattering cross section $\sigma(\phi) = 4\pi r^2 |u|^2 |A|^{-2}$. We obtain for the scalar cases (upper sign for soft cone, lower sign for hard cone)

$$(15) \quad \frac{\sigma(0)}{\pi a^2} = \frac{4}{n^2} \sin^2 \frac{\pi}{n} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} \mp \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{n} \right)^{-1} \right]^2$$

$$(16) \quad \frac{\sigma(\pi)}{\pi a^2} = (ka)^2$$

$$(17) \quad \frac{\sigma(\phi)}{\pi a^2} = \frac{2 \sin^2(\pi/n)}{n^2 \pi k a \sin \phi} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} \mp \left(\cos \frac{\pi}{n} - \cos \frac{3\pi - 2\phi}{n} \right)^{-1} \right]^2 \quad \gamma < \phi < \frac{\pi}{2}$$

$$(18) \quad \frac{\sigma(\phi)}{\pi a^2} = \frac{2 \sin^2(\pi/n)}{n^2 \pi k a \sin \phi} \left| e^{-i(2ka \sin \phi - \pi/4)} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} \mp \left(\cos \frac{\pi}{n} - \cos \frac{3\pi - 2\phi}{n} \right)^{-1} \right] \right.$$

$$\left. + e^{i(2ka \sin \phi - \pi/4)} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} \mp \left(\cos \frac{\pi}{n} - \cos \frac{3\pi + 2\phi}{n} \right)^{-1} \right] \right|^2, \quad 0 < \phi < \gamma$$

$$(19) \quad \frac{\sigma(\phi)}{\pi a^2} = \frac{2 \sin^2(\pi/n)}{n^2 \pi k a \sin \phi} \left| e^{-i(2ka \sin \phi - \pi/4)} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi - 2\phi}{n} \right)^{-1} \right] \right. \\ \left. + e^{i(2ka \sin \phi - \pi/4)} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{2\phi - \pi}{n} \right)^{-1} \right] \right|^2, \quad \frac{\pi}{2} < \phi < \pi$$

Equations (10) and (15) are not valid when $\gamma = \pi/2$, in which case the cone becomes a circular disk. In that case the right side of (10) must be replaced by that of (14) and that of (15) by the right side of (16).

In the electromagnetic case when $\phi \neq 0, \pi$ we may define the plane Γ containing the axis of the cone and the source (or an incident ray). Let us suppose that the incident wave is linearly polarized and that the incident electric field makes the angle Ψ with the normal to this plane. Then the component of the incident field normal to the edge at the points of diffraction is proportional to $\sin \Psi$ and that parallel to the edge is proportional to $\cos \Psi$. Thus the back scattered field component normal to the plane Γ is given by $u_{el}^S(r, \phi) \cos \Psi$ and that parallel to the plane Γ is $u_{el}^H(r, \phi) \sin \Psi$. Here u_{el}^S and u_{el}^H are the back scattered scalar fields. The superscript S refers to the soft cone and H to the hard cone. From these results we find that the back scattering cross section σ_{EM} for the electromagnetic case is given by

$$(20) \quad \sigma_{EM}(\phi) = \sigma_S(\phi) \cos^2 \Psi + \sigma_H(\phi) \sin^2 \Psi \quad \phi \neq 0, \pi$$

For axial incidence $\phi = 0, \pi$ the back scattered field $\vec{E}$ is given by

$$(21) \quad \vec{E} = \frac{1}{2}(u^S - u^H) \vec{E}_i$$

Here $\vec{E}_i$ is the incident field at the body while u^S and u^H denote the scalar back-scattered fields, which may be reflected, diffracted from edges, or diffracted around curved surfaces. The relation (21) holds for back scattering along the axis of any body of revolution. It can be proved by considering the fields on the various rays.^[14] Upon combining (21) with (10) we obtain

$$(22) \quad \frac{\sigma_{EM}^{(0)}}{\pi a^2} = \frac{4 \sin^2(\pi/n)}{n^2} \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{n} \right)^{-2}$$

Graphs of $\sigma(0)$ versus γ , based on (22) and (15), are shown in Figure 3. From (21) and (11) we find

$$(23) \quad \frac{\sigma_{EM}(\pi)}{\pi a^2} = (ka)^2$$

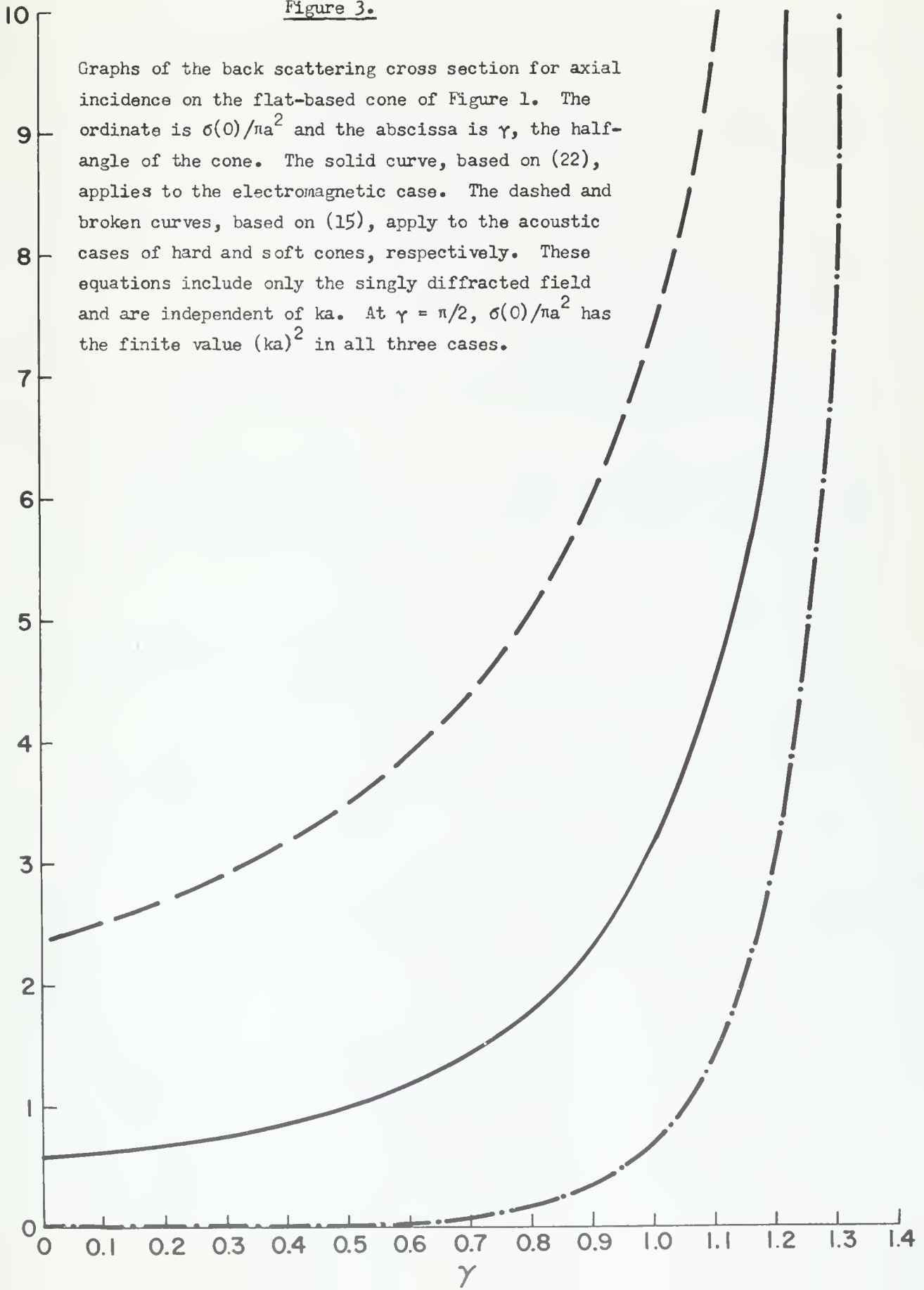
As in the scalar case, when $\gamma = \pi/2$ and the cone is a disk, the right side of (22) must be replaced by that of (23).

4. Flat based cone - double diffraction.

Some of the singly diffracted rays produced by the incident ray traverse the base of the cone, hit the opposite edge and produce doubly diffracted rays. Some of these rays return to the source, contributing

Figure 3.

$$\frac{\sigma}{\pi a^2}$$



to the back scattering. We shall now analyze this effect in the case of axial incidence upon the tip of the cone since in this case the effect of double diffraction is greater than that of tip diffraction.

The field on a ray singly diffracted at P_1 is given by the first term on the right side of (1). For a ray traversing the base of the cone $\rho_1 = -a$ and the diffraction coefficient D_1 is given by

$$(24) \quad D = \frac{e^{i\pi/4} \sin(\pi/n)}{n (2\pi k)^{1/2}} \left[\left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{2n} \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{2n} \right)^{-1} \right].$$

For the soft cone (24) shows that $D = 0$. This ray reaches the point P_2 on the opposite edge when $r_1 = 2a$. The field on this ray at P_2 is

$$(25) \quad u_{e1}(P_2) = D(2a)^{-1/2} e^{2ika - i\pi/2} u_1(P_1).$$

Similarly the field $u_{e1}(P_1)$ on a ray produced at P_2 is given by the right side of (25) with $u_1(P_1)$ replaced by $u_1(P_2)$, but with the same diffraction coefficient.

The field (25) may be considered to be the incident field which gives rise to a doubly diffracted ray. The doubly diffracted field $u_{e2}(P)$ at a point P which is near, but not on the axis will consist of two terms like those in (1), since two doubly diffracted rays pass through P . One of these comes from P_1 and another comes from P_2 . Thus we have

$$(26) \quad u_{e2}(P) = \left[r_1(1 + \rho_1^{-1} r_1) \right]^{-1/2} e^{ikr_1} D_3 u_{e1}(P_1) + \left[r_2(1 + \rho_1^{-1} r_2) \right]^{-1/2} e^{ikr_2} D_4 u_{e1}(P_2).$$

If the point P is far from the cone and near the axis, D_3 and D_4 are both equal to D which is given by (25).

We now proceed to simplify (26) exactly as we did in obtaining (10) from (1). In fact, by comparing (1) and (26) we see that they differ only by the factor $D_3 u_{e1}(P_1)/D_1 u_i(P_1)$. This is so because for $\gamma = 0, u_{e1}(P_1) = u_{e2}(P_2)$. Furthermore, for P far from the cone and near the axis, D_1 and D_2 are equal just as D_3 and D_4 are. As a consequence we can obtain the simplified form of (26) by multiplying (10) by the factor given above. Since $u_{e1}(P_1)$ as given by (25) contains $u_i(P_1)$, the factor becomes $D^2(2a)^{-1/2} e^{2ika - i\pi/2} (D_1)^{-1}$. In this way we obtain for the hard cone

$$(27) \quad u_{e2}^H(r, 0) = \frac{2\sin^2(\pi/n) a^{1/2} A e^{ikr} e^{2ika(\cot\gamma + 1) - i\pi/4}}{n^2 (\pi k)^{1/2} r} \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{2n} \right)^{-2}.$$

For the soft cone $D = 0$ and therefore

$$(28) \quad u_{e2}^S(r, 0) = 0.$$

Let us now combine the singly and doubly diffracted back scattered fields.

For the hard cone (10) and (27) yield

$$(29) \quad u_{e1}^H(r, 0) + u_{e2}^H(r, 0) = \frac{a A \sin(\pi/n) e^{ik(r + 2a \cot\gamma)}}{n r} \left[\left(\cos \frac{\pi}{n} - 1 \right)^{-1} + \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{2n} \right)^{-1} \right. \\ \left. + \frac{2\sin(\pi/n) e^{2ika - i\pi/4}}{n (\pi k a)^{1/2}} \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{2n} \right)^{-2} \right]..$$

The back scattering cross section $\sigma_H(0)$ is now $4\pi r^2 A^{-2}$ multiplied by the square of the absolute value of the right hand side in (29). For the soft cone (10) and (15) still apply since $u_{e2}^S(r, 0) = 0$.

For the electromagnetic case we insert (10) for u^S and (29) for u^H into (21). Then if $\vec{A}$ denotes the vector amplitude of the incident field at the cone, we obtain

$$(30) \quad \vec{E}_{e1}(r,0) + \vec{E}_{e2}(r,0) = - \frac{a\vec{A} \sin(\pi/n) e^{ik(r + 2a \cot \gamma)}}{nr} \left[\left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{n} \right)^{-1} + \frac{\sin(\pi/n) e^{2ika - i\pi/4}}{n(\pi ka)^{1/2}} \left(\cos \frac{\pi}{n} - \cos \frac{3\pi}{2n} \right)^{-2} \right] .$$

The back scattering cross section $\sigma_{EM}(0)$ is $4\pi r^2 |\vec{A}|^{-2}$ multiplied by the square of the absolute value of the right hand side in (30). Graphs of $\sigma(0)$ versus ka , based on (29) and (30), are shown in Figure 4.

5. Cone with a spherical base

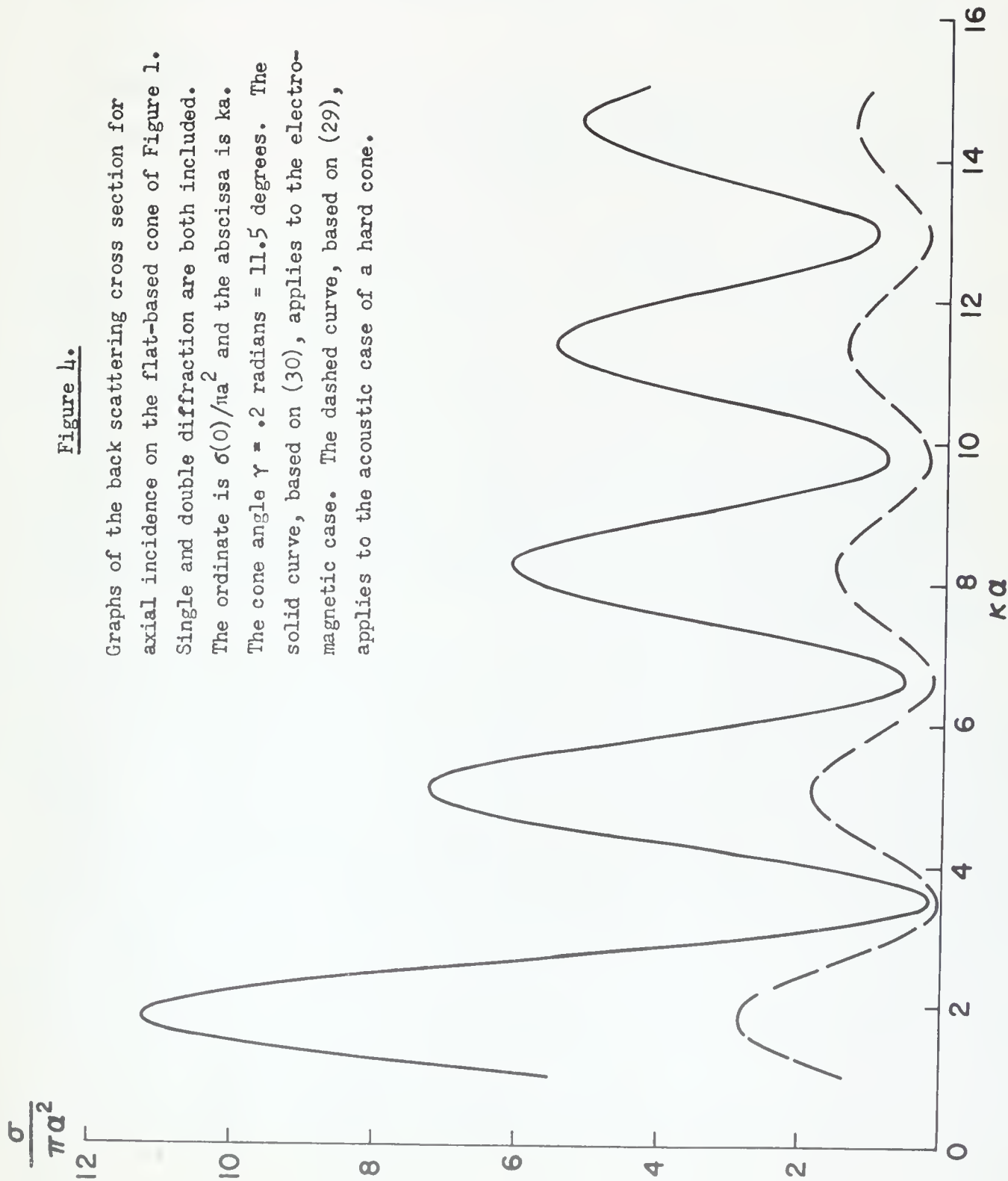
Let us now consider back scattering from the cone of Figure 2. When the direction of incidence lies in the range $\pi/2 - \gamma < \phi \leq \pi$ some incident ray hits the cone normally and is reflected back to the source. The back scattering cross section for such directions is then given by the well-known geometrical optica formula

$$(31) \quad \sigma_S = \sigma_H = \sigma_{EM} = \pi a^2 \quad \pi/2 - \gamma < \phi \leq \pi .$$

When the direction of incidence lies in the range $0 \leq \phi < \pi/2 - \gamma$, only a tip diffracted ray and one or more surface diffracted rays return to the source. In the range $0 \leq \phi \leq \gamma$ infinitely many surface diffracted rays return to the source, and they form an axial caustic extending in the direction of the source. The field on this caustic is the same as the diffracted part of the back-scattered field from a sphere, if rays which make complete revolutions around the sphere are excluded. The latter field is given in reference [15] equations (95) and (96). If we add this field to that on the tip

Figure 4.

Graphs of the back scattering cross section for axial incidence on the flat-based cone of Figure 1. Single and double diffraction are both included. The ordinate is $\sigma(0)/\pi a^2$ and the abscissa is ka . The cone angle $\gamma = .2$ radians = 11.5 degrees. The solid curve, based on (30), applies to the electromagnetic case. The dashed curve, based on (29), applies to the acoustic case of a hard cone.



diffracted ray we obtain in the scalar case

$$(32) \quad u(r, \phi) = \frac{aAe^{ikr}}{2r} \left[2C(\phi)(ka)^{-1} + G(ka)^{1/3} e^{-\tau(ka)^{1/3}} e^{ika(\pi+2\text{csc}\gamma\cos\phi)} \right], \quad 0 \leq \phi \leq \gamma.$$

In (32) $C(\phi)$ is the tip diffraction coefficient for back scattering which depends upon ϕ and upon whether the cone is soft or hard. The constants G and τ are also different in the two cases and are given by

$$(33) \quad G_S = -4\pi^2 \epsilon^{-4/3} [A'(q_S)]^{-2} e^{i\pi/3}, \quad \tau_S = e^{-i\pi/6} \epsilon^{-1/3} q_S \pi, \quad A(q_S) = 0 \quad \text{SOFT CONE}$$

$$(34) \quad G_H = -2\pi^2 \epsilon^{-1/3} q_H^{-1} [A(q_H)]^{-2} e^{i\pi/3}, \quad \tau_H = e^{-i\pi/6} \epsilon^{-1/3} q_H \pi, \quad A'(q_H) = 0 \quad \text{HARD CONE}$$

In these equations the subscripts S and H refer to the soft and hard cones. The function $A(q)$ is the Airy function. The roots q_S and q_H are given in reference [15]. The tip diffraction coefficient has been determined by L. B. Felsen and the others referred to in the introduction.

In the electromagnetic case we may use (21) to obtain the contribution to $\vec{E}$ from surface diffracted rays in the range $0 \leq \phi \leq \gamma$. This is so because for these rays in this range the rear hemisphere accounts for the back scattering, and it is axially symmetric about any axis in this range. The field back scattered from the tip must be computed with the aid of the tip diffraction coefficient matrix which we will also denote by $C(\phi)$. Thus we have

$$(35) \quad \vec{E}(r, \phi) = \frac{a}{2r} e^{ikr} \left[2(ka)^{-1} C(\phi) + \frac{(ka)^{1/3}}{2} e^{ika(\pi+2\text{csc}\gamma\cos\phi)} \right. \\ \left. \times \begin{pmatrix} G_S e^{-\tau_S(ka)^{1/3}} & -G_H e^{-\tau_H(ka)^{1/3}} \end{pmatrix} \right] \vec{E}_1, \quad 0 \leq \phi \leq \gamma$$

This result can be simplified by omitting the term $G_S \exp[-\tau_S(ka)^{1/3}]$ since it is much smaller than the term $G_H \exp[-\tau_H(ka)^{1/3}]$.

6. Conclusion.

On the basis of the preceding considerations it is possible to draw some conclusions about shaping an object to minimize its back-scattering cross section σ . These conclusions are

a) The object should not have any flat, cylindrical or conical surfaces which might be hit normally by an incident wave. Such surfaces can only be hit normally from a narrow range of directions. Although they yield very strong reflections within this range, they yield correspondingly small return in all other directions. For example an object with an ogival nose instead of a conical one yields a smaller reflection when it is hit normally, but it can be hit normally from a greater range of directions. The decision about whether to include flat, cylindrical or conical surfaces and thus permit big reflections in a small range of directions and very little in other directions, or to exclude them and use only doubly curved surfaces which yield a somewhat smaller reflection over a larger range of directions, must be based on operational considerations.

If only a limited range of directions of incidence need be considered then:

b) The object should have no surface which can be hit normally from the range of incidence directions under consideration. This is the case for a finite cone when the incidence directions are confined to the neighborhood of the axis.

c) If the object does have a surface which can be hit normally, that surface should be as curved as possible. Thus if the object is a cone with a rounded tip and if the incidence directions are nearly axial, the tip should

have as small a radius as possible.

d) The object should have no edge which can be hit normally by an incident ray. Thus if the object is a finite cone with a flat or curved rear surface, the conical and rear surfaces should join smoothly and not meet at a sharp edge. Whether a sharply curved surface behaves as an edge or not depends upon the relative sizes of its radius of curvature and the wavelength. The larger the radius compared to the wavelength, the less the surface behaves like an edge. Of course rounding off such an edge diminishes the back-scattering from axial and nearly axial directions, but it increases back-scattering in certain directions near the rear from which reflection from the round edge now becomes possible.

Suppose the object is a finite cone with incidence directions which are nearly axial. Then

e) It should have a rounded rather than a flat rear surface. This diminishes the contribution of the surface rays which traverse the rear surface.

f) The rear surface should not be axially symmetric. This will eliminate the caustic of the diffracted rays on the axis, in the case of axial incidence. Thus it will significantly cut down the cross section for such incidence.

g) The rear surface should probably have some sharp edges or conical tips. This will help to interrupt the surface rays on the rear surface and cause them to radiate most of their energy away from the surface.

Suggestions similar to some of these have been made by S. Silver and by K.M. Siegel.

References

- [1] Keller, J.B. - A geometrical theory of diffraction, Calculus of Variations and Its Applications, Proc. of Symp. in Appl. Math. Vol. 8, pp. 27-52, Mc Graw-Hill, New York 1958.
- [2] - Jour. App. Phys. 28, 426-444 (1957).
- [3] Carslaw, H.S. Math. Ann. 75, 133-147 (1914).
- [4] Hansen, W.W. and Schiff, L.I. Microwave Laboratory, Stanford Univ., Quarterly Reports 1,2,3,4, Nov. 1947 - Sept. 1948, Contract W28-099-333.
- [5] Schensted, C.E. Application of summation techniques to the radar cross-section of a cone, Symposium on Microwave Optics, Mc Gill Univ., Montreal, Canada, June 1953.
- [6] Siegel, K.M., Crispin, J.W. and Schensted, C.E. Jour. App. Phys. 26, 309-313 (1955).
- [7] Felsen, L.B. Back scattering from wide-angle and narrow-angle cones, Symposium on Microwave Optics, Mc Gill Univ., Montreal, Canada, June 1953.
- [8] - Jour. App. Phys. 26, 138-151 (1955).
- [9] - IRE Transactions PGAP Vol. AP-5, 121-129 (1957).
- [10] - Back scattering from a semi-infinite cone, Microwave Research Inst., Polytechnic Inst. of Brooklyn, Memo. 43, R-675-58, PIB-603, July 1958.
- [11] Spencer, R.C. Back scattering from conducting surfaces, Air Force Cambridge Research Laboratories, Report CRL-E5070, April 1951.
- [12] Siegel, K.M., Alperin, H.A., Bonkowski, R.R., Crispin, J.W., Moffett, A.L., Schensted, C.E. and Schensted, I.V. Jour. App. Phys. 26, 297-305 (1955)

- [13] Siegel, K.W. Far field scattering from bodies of revolution;
URSI 12th General Assembly, Boulder, Colorado (1957).

- [14] Levy, B.R. and Keller, J.B. Diffraction by a spheroid; N.Y.U., Inst. Math.
Sci., Div. EM Research, Report No. EM-130,
April, 1959.

- [15] - ' Comm. Pure and Appl. Math.; 12, 159 (1959).

DISTRIBUTION LIST FOR RESEARCH REPORTS

Contract No. AF 19(604)1717

(ONE copy unless otherwise noted)

Commander Air Research and Development Command Andrews Air Force Base Washington 25, D. C. Attn: Major E. Wright, RDTCC	Director Evans Signal Laboratory Belmar, New Jersey Attn: Mr. O. C. Woodyard	(5) Hq. Air Force Cambridge Research Center Laurence G. Hanscom Field Bedford, Mass. Attn: CRRD
Director of Resident Training 3380th Technical Training Group Keesler Air Force Base, Mississippi Attn: OA-3011 Course	U. S. Army Signal Engineering Labs. Evans Signal Laboratory Belmar, New Jersey Attn: Technical Document Center	Director, Avionics Division (AV) Bureau of Aeronautics Department of the Navy Washington 25, D. C.
Director Air University Library Maxwell Air Force Base, Alabama	Massachusetts Institute of Technology Signal Corps Liaison Officer Cambridge 39, Mass. Attn: A. D. Bedrosian, Room 26-131	Chief, Bureau of Ships Department of the Navy Washington 25, D. C. Attn: Mr. E. Johnston, Code 833E
Commander Air Force Missile Test Center Petrick Air Force Base, Florida Attn: MTE - for classified documents Attn: MU-411, Technical Library - for unclassified documents	Commanding General, SIGFM/EL-PC U. S. Army Signal Engineering Labs. Fort Monmouth, New Jersey Attn: Dr. Horet H. Kedesdy Deputy Chief, Chem-Physics Branch	Commander U. S. Naval Air Missile Test Center Point Mugu, California Attn: Code 366
Tactical Air Group Directorate of Research and Development DCS/D Headquarters, USAF Washington, D. C. Attn: Major R. L. Stell	Commander Army Rocket and Guided Missile Agency Redstone Arsenal, Alabama Attn: Technical Library, ORDXR-OTL	U. S. Naval Ordnance Laboratory White Oak Silver Spring 19, Maryland Attn: The Library
Director, Communications and Electronics Hq. U. S. Air Force Washington 25, D. C. Attn: AFOAC S/E	Commanding General U. S. Army Signal Engineering Labs. Fort Monmouth, New Jersey Attn: SIGFM/EL-AT	Commander U. S. Naval Ordnance Test Station China Lake, California Attn: Code 753
Commander Wright Air Development Center Wright-Patterson Air Force Base, Ohio Attn: WCLRS-6, Mr. Portune	Department of the Army Office of the Chief Signal Officer Washington 25, D. C. Attn: SIGRD-7	Librarian U. S. Naval Postgraduate School Monterey, California
Wright Air Development Center Wright-Patterson Air Force Base, Ohio Flight Research Laboratory Research Division Attn: WCRRA	Office of Chief Signal Officer Engineering and Technical Division Washington 25, D. C. Attn: SIGNET-5	Air Force Development Field Representative Naval Research Laboratory Washington 25, D. C. Attn: Code 1072
Commander Wright Air Development Center Wright-Patterson Air Force Base, Ohio Attn: N. Draganjac, WCLNQ-4	Guided Missile Fuze Library Diamond Ordnance Fuze Laboratories Washington 25, D. C. Attn: R. D. Hatcher, Chief Microwave Development Section	Director U. S. Naval Research Laboratory Washington 25, D. C. Attn: Code 2027
Commander Wright Air Development Center Wright-Patterson Air Force Base, Ohio Attn: Mr. Paul Springer, WCLRE-5	(10) Armed Services Technical Information Agency Arlington Hall Station Arlington 12, Virginia	Dr. J. I. Bohnert, Code 5210 U. S. Naval Research Laboratory Washington 25, D. C. (Unclassified only) Classified to be sent to: Director U. S. Naval Research Laboratory Attn: Code 5200 Washington 25, D. C.
Commander Air Technical Intelligence Center Wright-Patterson Air Force Base, Ohio Attn: AFCIN-4B1a	(2) Library Boulder Laboratories National Bureau of Standards Boulder, Colorado	Commanding Officer and Director U. S. Navy Underwater Sound Laboratory Fort Trumbull, New London, Connecticut
Commander Rome Air Development Center Griffiss Air Force Base, New York Attn: RCSSTL-1	National Bureau of Standards Department of Commerce Washington 25, D. C. Attn: Mr. A. G. McNish	Chief of Naval Research Department of the Navy Washington 25, D. C. Attn: Code h27
Commander Rome Air Development Center Griffiss Air Force Base, New York Attn: Mr. Donald Dakan, RCUE	National Bureau of Standards Department of Commerce Washington 25, D. C. Attn: Gustave Shapiro, Chief Engineering Electronics Section Electricity and Electronics Div.	Commanding Officer and Director U. S. Navy Electronics Laboratory (Library) San Diego 52, California
Commander Rome Air Development Center (ARDC) Griffiss Air Force Base, New York Attn: Dr. John S. Burgess, RCE	(2) Office of Technical Services Department of Commerce Washington 25, D. C. Attn: Technical Reports Section (Unclassified only)	Chief, Bureau of Ordnance Department of the Navy Washington 25, D. C. Attn: Code Ad3
Commander Air Force Missile Development Center Holloman Air Force Base, New Mexico Attn: HDOLL, Technical Library	Director National Security Agency Washington 25, D. C. Attn: R/D (331)	Chief, Bureau of Ordnance Department of the Navy Washington 25, D. C. Attn: Fire Control Branch (ReSh)
Director U. S. Army Ordnance Ballistic Research Laboratories Aberdeen Proving Ground, Maryland Attn: Ballistic Measurements Laboratory	(2) Hq. Air Force Cambridge Research Center Laurence G. Hanscom Field Bedford, Mass. Attn: CROTLR-2 - P. Condon	Department of the Navy Bureau of Aeronautics Technical Data Division, Code h106 Washington 25, D. C.
Ballistic Research Laboratories Aberdeen Proving Ground, Maryland Attn: Technical Information Branch	(5) Hq. Air Force Cambridge Research Center Laurence G. Hanscom Field Bedford, Mass. Attn: CROTLR - J. Armstrong	Chief, Bureau of Ships Department of the Navy Washington 25, D. C.

Commanding Officer
U. S. Naval Air Development Center
Johnsville, Pennsylvania
Attn: NADC Library

Commander
U. S. Naval Air Test Center
Patuxent River, Maryland
Attn: ET-315, Antenna Branch

Director
Naval Ordnance Laboratory
Corona, California

Commanding Officer
U. S. Naval Ordnance Laboratory
Corona, California
Attn: Mr. W. Rorenstein, Division 72

Airborne Instruments Laboratory, Inc.
160 Old Country Road
Mineola, New York
Attn: Dr. E. G. Fubini, Director
Research and Engineering Division

Aircom, Inc.
354 Main Street
Winthrop, Mass.

American Machine and Foundry Company
Electronics Division
1085 Commonwealth Avenue
Boston 15, Mass.
Attn: Mrs. Rita Moravcsik, Librarian

Andrew Alford, Consulting Engineers
299 Atlantic Avenue
Boston 10, Mass.

Avion Division
ACF Industries, Inc.
800 No. Pitt Street
Alexandria, Virginia
Attn: Library

Battelle Memorial Institute
505 King Avenue
Attn: Wayne E. Rife, Project Leader
Electrical Engineering Division
Columbus 1, Ohio

Bell Aircraft Corporation
Post Office Box One
Buffalo 5, New York
Attn: Eunice P. Hazelton, Librarian

Bell Telephone Laboratories, Inc.
Whippany Laboratory
Whippany, New Jersey
Attn: Technical Information Library

Pacific Division
Bendix Aviation Corporation
11600 Sherman Way
North Hollywood, California
Engineering Library
Attn: Peggie Robinson, Librarian

Bendix Radio Division
Bendix Aviation Corp.
E. Joppa Road
Towson 4, Maryland
Attn: Dr. D. M. Allison, Jr.
Director Engineering and Research

Boeing Airplane Company
Pilotless Aircraft Division
P.O. Box 3707
Seattle 24, Washington
Attn: R.R. Barber, Library Supervisor

Boeing Airplane Company
Wichita Division Engineering Library
Wichita 1, Kansas
Attn: Kenneth C. Knight, Librarian

Boeing Airplane Company
Seattle Division
Seattle 14, Washington
Attn: E.T. Allen, Library Supervisor

Bjorksten Research Labs, Inc.
P. O. Box 265
Madison, Wisconsin
Attn: Mrs. Fern B. Korsgard

Convair, A Division of General Dynamics
Corp.
Fort Worth, Texas
Attn: K.G. Brown, Division Research
Librarian

Convair, A Division of General Dynamics
Corp.
San Diego 12, California
Attn: Mrs. Dora B. Burke,
Engineering Librarian

Cornell Aeronautical Laboratory, Inc.
4455 Genesee Street
Buffalo 21, New York
Attn: Librarian

Dalmo Victor Company
A Division of Textron, Inc.
1515 Industrial Way
Belmont, California
Attn: Mary Ellen Addams,
Technical Librarian

Dorne and Margolin, Inc.
29 New York Avenue
Westbury, Long Island, N. Y.

Douglas Aircraft Company, Inc.
P.O. Box 200
Long Beach 1, California
Attn: Engineering Library (C-250)

Douglas Aircraft Co., Inc.
827 Lapham Street
El Segundo, California
Attn: Engineering Library

Douglas Aircraft Company, Inc.
3000 Ocean Park Boulevard
Santa Monica, California
Attn: P.T. Cline
Eq. Sec. Reference Files,
Eq. Eng. A250

Douglas Aircraft Company, Inc.
2000 North Memorial Drive
Tulsa, Oklahoma
Attn: Engineering Library, D-250

Electronics Communication, Inc.
1830 York Road
Timonium, Maryland

Emerson and Cuming, Inc.
869 Washington Street
Canton, Mass.
Attn: Mr. W. Cuming

Emerson Electric Mfg. Co.
8100 West Florissant Avenue
St. Louis 21, Missouri
Attn: Mr. E.R. Brealin, Librarian

Sylvania Elec. Prod. Inc.
Electronic Defense Laboratory
P.O. Box 205 - (Uncl)
Mountain View, California
Attn: Library

Fairchild Aircraft Division
Fairchild Eng. and Airplane Corp.
Hagerstown, Maryland
Attn: Library

Farnsworth Electronics Company
3700 East Pontiac Street
Fort Wayne 1, Indiana
Attn: Technical Library

Federal Telecommunication Labs.
500 Washington Avenue
Nutley 10, New Jersey
Attn: Technical Library

The Gabriel Electronics
Division of the Gabriel Company
135 Crescent Road
Needham Heights 94, Mass.
Attn: Mr. Steven Galagan

General Electric Advanced Electronics Center
Cornell University
Ithaca, New York
Attn: J. B. Travis

General Electric Company
Electronics Park
Syracuse, New York
Attn: Documents Library, B. Fletcher
Building 3-143A

General Precision Laboratory, Inc.
63 Bedford Road
Pleasantville, New York
Attn: Mrs. Mary G. Herbst, Librarian

Goodyear Aircraft Corp.
1210 Massillon Road
Akron 15, Ohio
Attn: Library D/120 Plant A

Granger Associates
Electronic Systems
966 Commercial Street
Palo Alto, California
Attn: John V. N. Granger, President

Grumman Aircraft Engineering Corporation
Bethpage, Long Island, N. Y.
Attn: Mrs. A. M. Gray, Librarian
Engineering Library, Plant No. 5

The Hallicrafters Company
4401 West 5th Avenue
Chicago 24, Illinois
Attn: LaVerne LaGioia, Librarian

Hoffman Laboratories, Inc.
3761 South Hill Street
Los Angeles 7, California
Attn: Engineering Library

Hughes Aircraft Company
Antenna Department
Microwave Laboratory
Building 12, Room 2617
Culver City, California
Attn: M. D. Adcock

Hughes Aircraft Company
Florence and Teale Streets
Culver City, California
Attn: Dr. L.C. Van Atta, Associate Director
Research Labs.

Hycon Eastern, Inc.
75 Cambridge Parkway
Cambridge, Mass.
Attn: Mrs. Lois Seulowitz
Technical Librarian

International Business Machines Corp.
Military Products Division
590 Madison Avenue
New York 33, New York
Attn: Mr. C.F. McElwain, General Manager

International Business Machines Corp.
Military Products Division
Owego, New York
Attn: Mr. D. I. Marr, Librarian
Department 459

International Resistance Company
401 N. Broad Street
Philadelphia 8, Pa.
Attn: Research Library

Jansky and Bailey, Inc.
1339 Wisconsin Avenue, N. W.
Washington 7, D. C.
Attn: Mr. Delmer C. Ports

Dr. Henry Jasik, Consulting Engineer
298 Shames Drive
Brush Hollow Industrial Park
Westbury, New York

Electromagnetic Research Corporation
711 14th Street, N. W.
Washington 5, D. C.

Lockheed Aircraft Corporation
2555 N. Hollywood Way
California Division Engineering Library
Department 72-75, Plant A-1, Bldg. 63-1
Burbank, California
Attn: N. C. Harnois

The Martin Company
P. O. Box 179
Denver 1, Colorado
Attn: Mr. Jack McCormick

The Glenn L. Martin Company
Baltimore 3, Maryland
Attn: Engineering Library
Antenna Design Group

Maryland Electronic Manufacturing Corp.
5009 Calvert Road
College Park, Maryland
Attn: Mr. R. Warren Cooper

Mathematical Reviews
190 Hope Street
Providence 6, Rhode Island

The W. L. Maxson Corporation
460 West 34th Street
New York, N. Y.
Attn: Miss Dorothy Clark

McDonnell Aircraft Corporation
Lambert Saint-Louis Municipal Airport
Box 516, St. Louis 3, Missouri
Attn: R. D. Detrich, Engineering Library

McMillan Laboratory, Inc.
Brownville Avenue
Ipswich, Massachusetts
Attn: Security Officer, Document Room

Melpar, Inc.
3000 Arlington Boulevard
Falls Church, Virginia
Attn: Engineering Technical Library

Microwave Development Laboratory
90 Broad Street
Babson Park 57, Massachusetts
Attn: N. Tucker, General Manager

Microwave Radiation Company Inc.
19223 South Hamilton Street
Gardena, California
Attn: Mr. Morris J. Ehrlich, President

Chance Vought Aircraft, Inc.
9314 West Jefferson Street
Dallas, Texas
Attn: Mr. H. S. White, Librarian

Northrop Aircraft, Inc.
Hawthorne, California
Attn: Mr. E. A. Freitas, Library Dept 3145
1001 E. Broadway

Remington Rand Univ. - Division of Sperry
Rand Corporation
1900 West Allegheny Avenue
Philadelphia 29, Pennsylvania
Attn: Mr. John F. McCarthy
R and D Sales and Contracts

North American Aviation, Inc.
12214 Lakewood Boulevard
Downey, California
Attn: Engineering Library 495-115

North American Aviation, Inc.
Los Angeles International Airport
Los Angeles 45, California
Attn: Engineering Technical File

Page Communications Engineers, Inc.
710 Fourteenth Street, Northwest
Washington 5, D. C.
Attn: Librarian

Philco Corporation Research Division
Branch Library
4700 Wissachickon Avenue
Philadelphia 44, Pa.
Attn: Mrs. Dorothy S. Collins

Pickard and Burns, Inc.
240 Highland Avenue
Needham 94, Mass.
Attn: Dr. J. T. DaBettencourt

Polytechnic Research and Development
Company, Inc.
202 Tillary Street
Brooklyn 1, New York
Attn: Technical Library

Radiation Engineering Laboratory
Main Street
Maynard, Mass.
Attn: Dr. John Ruzé

Radiation, Inc.
P. O. Drawer 37
Melbourne, Florida
Attn: Technical Library, Mr. M. L. Cox

Radio Corp. of America
RCA Laboratories
Rocky Point, New York
Attn: P. S. Carter, Lab. Library

RCA Laboratories
David Sarnoff Research Center
Princeton, New Jersey
Attn: Miss Fern Cloak, Librarian
Research Library

Radio Corporation of America
Defense Electronic Products
Building 10, Floor 7
Camden 2, New Jersey
Attn: Mr. Harold J. Schrader
Staff Engineer, Organization
of Chief Technical
Administrator

(2) The Ramo-Wooldridge Corporation
P.O. Box 45453 Airport Station
Los Angeles 45, California
Attn: Margaret C. Whitnah,
Chief Librarian

Hoover Microwave Co.
9592 Baltimore Avenue
College Park, Maryland

Director, USAF Project RAND
Via: Air Force Liaison Office
The Rand Corporation
1700 Main Street
Santa Monica, California

Rantec Corporation
Calabasas, California
Attn: Grace Keener, Office Manager

Raytheon Manufacturing Company
Missile Systems Division
Bedford, Mass.
Attn: Mr. Irving Goldstein

Raytheon Manufacturing Company
Wayland Laboratory, State Road
Wayland, Mass.
Attn: Mr. Robert Borts

Raytheon Manufacturing Company
Wayland Laboratory
Wayland, Mass.
Attn: Miss Alice G. Anderson,
Librarian

Republic Aviation Corporation
Farmingdale, Long Island, N. Y.
Attn: Engineering Library

Thru: Air Force Plant Representative
Republic Aviation Corp.
Farmingdale, Long Island, N.Y.

Rheem Manufacturing Company
9236 East Hall Road
Downey, California
Attn: J. C. Joerger

Trans-Tech, Inc.
P. O. Box 346
Frederick, Maryland

Ryan Aeronautical Company
Lindbergh Field
San Diego 12, California
Attn: Library - unclassified

Sage Laboratories
159 Linden Street
Wellesley 81, Mass.

Sanders Associates
95 Canal Street
Nashua, New Hampshire
Attn: N. R. Wild, Librarian

Sandia Corporation, Sandia Base
P.O. Box 5800, Albuquerque, New Mexico
Attn: Classified Document Division

Sperry Gyroscope Company
Great Neck, Long Island, New York
Attn: Florence W. Turnbull, Engr. Librarian

Stanford Research Institute
Menlo Park, California
Attn: Library, Engineering Division

Sylvania Electric Products, Inc.
100 First Avenue
Waltham 54, Mass.
Attn: Charles A. Thornhill, Report Librarian
Waltham Laboratories Library

Systems Laboratories Corporation
14852 Ventura Boulevard
Sherman Oaks, California
Attn: Donald L. Margerum

TRG, Inc.
17 Union Square West
New York 3, N. Y.
Attn: M. L. Henderson, Librarian

A. S. Thomas, Inc.
161 Devonshire Street
Boston 10, Mass.
Attn: A. S. Thomas, President

Bell Telephone Laboratories
Murray Hill
New Jersey

Chu Associates
P. O. Box 387
Whitcomb Avenue
Littleton, Mass.

Microwave Associates, Inc.
Burlington, Mass.

Raytheon Manufacturing Company
Missile Division
Hartwell Road
Bedford, Mass.

Radio Corporation of America
Aviation Systems Laboratory
225 Crescent Street
Waltham, Mass.

Lockheed Aircraft Corporation
Missile Systems Division Research Library
Box 504, Sunnyvale, California
Attn: Miss Eva Lou Robertson,
Chief Librarian

The Rand Corporation
1700 Main Street
Santa Monica, California
Attn: Dr. W. C. Hoffman

Commander
AF Office of Scientific Research
Air Research and Development Command
14th Street and Constitution Avenue
Washington, D. C.
Attn: Mr. Otting, SRY

Westinghouse Electric Corp.
Electronics Division
Friendship Int'l Airport Box 746
Baltimore 3, Maryland
Attn: Engineering Library

Wheeler Laboratories, Inc.
122 Cutter Mill Road
Great Neck, New York
Attn: Mr. Harold A. Wheeler

Zenith Plastics Co.
Box 91
Gardena, California
Attn: Mr. S. S. Oleesky

Library Geophysical Institute
of the University of Alaska
College
Alaska

University of California
Berkeley 4, California
Attn: Dr. Samuel Silver,
Prof. Engineering Science
Division of Elec. Eng. Electronics
Research Lab.

University of California
Electronics Research Lab.
332 Cory Hall
Berkeley 4, California
Attn: J. R. Whinnery

California Institute of Technology
Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, California
Attn: Mr. I. E. Newlan

California Institute of Technology
1201 E. California Street
Pasadena, California
Attn: Dr. C. Papas

Carnegie Institute of Technology,
Schenley Park
Pittsburgh 13, Pennsylvania
Attn: Prof. A. E. Heins

Cornell University
School of Electrical Engineering
Ithaca, New York
Attn: Prof. G. C. Dalman

University of Florida
Department of Electrical Engineering
Gainesville, Florida
Attn: Prof. M. H. Latour, Librarian

Library
Georgia Institute of Technology
Engineering Experiment Station
Atlanta, Georgia
Attn: Mrs. J.H. Crosland, Librarian

Harvard University
Technical Reports Collection
Gordon McKay Library, 303A Pierce Hall
Oxford Street, Cambridge 38, Mass.
Attn: Mrs. E.L. Hufschmidt, Librarian

Harvard College Observatory
60 Garden Street
Cambridge 39, Mass.
Attn: Dr. Fred L. Whipple

University of Illinois
Documenta Division Library
Urbana, Illinois

University of Illinois
College of Engineering
Urbana, Illinois
Attn: Dr. P. E. Moyes, Department of
Electrical Engineering

The Johns Hopkins University
Homewood Campus
Department of Physics
Baltimore 18, Maryland
Attn: Dr. Donald E. Kerr

Sandia Corporation
Attn: Organization 1423
Sandia Base
Albuquerque, New Mexico

Applied Physics Laboratory
The Johns Hopkins University
8621 Georgia Avenue
Silver Spring, Maryland
Attn: Mr. George L. Seisla

Massachusetts Institute of Technology
Research Laboratory of Electronics
Room 20B-221
Cambridge 39, Massachusetts
Attn: John H. Hewitt

Massachusetts Institute of Technology
Lincoln Laboratory
P. O. Box 73
Lexington 73, Mass.
Attn: Document Room A-229

University of Michigan
Electronic Defense Group
Engineering Research Institute
Ann Arbor, Michigan
Attn: J. A. Boyd, Supervisor

University of Michigan
Engineering Research Institute
Radiation Laboratory
Attn: Prof. K. M. Siegel
912 N. Main St.,
Ann Arbor, Michigan

University of Michigan
Engineering Research Institute
Willow Run Laboratories
Willow Run Airport
Ypsilanti, Michigan
Attn: Librarian

University of Minnesota
Minneapolis 14, Minnesota
Attn: Mr. Robert H. Stumm, Librarian

Northwestern University
Microwave Laboratories
Evanston, Illinois
Attn: R. E. Beam

Ohio State University Research Found.
Ohio State University
Columbus 10, Ohio
Attn: Dr. T.E. Tice
Dept. of Elec. Engineering

The University of Oklahoma
Research Institute
Norman, Oklahoma
Attn: Prof. C. L. Farrar, Chairman
Electrical Engineering

Polytechnic Institute of Brooklyn
Microwave Research Institute
55 Johnson Street
Brooklyn, New York
Attn: Dr. Arthur A. Oliner

Polytechnic Institute of Brooklyn
Microwave Research Institute
55 Johnson Street
Brooklyn, New York
Attn: Mr. A. E. Laemmel

Syracuse University Research Institute
Collendale Campus
Syracuse 10, New York
Attn: Dr. C. S. Grove, Jr.
Director of Engineering Research

The University of Texas
Elec. Engineering Research Laboratory
P. O. Box 8026, University Station
Austin 12, Texas
Attn: Mr. John R. Gerhardt
Assistant Director

The University of Texas
Defense Research Laboratory
Austin, Texas
Attn: Claude W. Horton, Physics Library

University of Toronto
Department of Electrical Engineering
Toronto, Canada
Attn: Prof. G. Sinclair

Lowell Technological Institute
Research Foundation
P. O. Box 709, Lowell, Mass.
Attn: Dr. Charles R. Mingins

University of Washington
Department of Electrical Engineering
Seattle 5, Washington
Attn: G. Held, Associate Professor

Stanford University
Stanford, California
Attn: Dr. Chodorow
Microwave Laboratory

Physical Science Laboratory
New Mexico College of Agriculture
and Mechanic Arts
State College, New Mexico
Attn: Mr. H. W. Haas

Brown University
Department of Electrical Engineering
Providence, Rhode Island
Attn: Dr. C. M. Angulo

Case Institute of Technology
Cleveland, Ohio
Attn: Prof. S. Seeley

Columbia University
Department of Electrical Engineering
Morningstar Heights
New York, N. Y.
Attn: Dr. Schlesinger

McGill University
Montreal, Canada
Attn: Prof. G. A. Wootton
Director, The Eaton Electronics
Research Lab.

Purdue University
Department of Electrical Engineering
Lafayette, Indiana
Attn: Dr. Schultz

The Pennsylvania State University
Department of Electrical Engineering
University Park, Pennsylvania

University of Pennsylvania
Institute of Cooperative Research
3400 Walnut Street
Philadelphia, Pennsylvania
Attn: Dept. of Electrical Engineering

University of Tennessee
Ferris Hall
W. Cumberland Avenue
Knoxville 16, Tennessee

University of Wisconsin
Department of Electrical Engineering
Madison, Wisconsin
Attn: Dr. Schaibe

University of Seattle
Department of Electrical Engineering
Seattle, Washington
Attn: Dr. D. K. Reynolds

Wayne University
Detroit, Michigan
Attn: Prof. A. F. Stevenson

Electronics Research Laboratory
Illinois Institute of Technology
3300 So. Federal Street
Chicago 16, Illinois
Attn: Dr. Lester C. Peach
Research Engineer

Advisory Group on Electronic Parts
Room 103
Moore School Building
200 South 33rd Street
Philadelphia 4, Pennsylvania

Ionosphere Research Laboratory Pennsylvania State College State College, Pennsylvania ATTN: Professor A. H. Waynick, Director	Dr. Rodman Doll 311 W. Cross Street Ypsilanti, Michigan	Smyth Research Associates 3555 Aero Court San Diego 3, California ATTN: Dr. John B. Smyth
Institute of Mathematical Sciences 25 Waverly Place New York 3, New York ATTN: Librarian	California Inst. of Technology Pasadena, California ATTN: Mr. Calvin Wilcox	Electrical Engineering California Institute of Technology Pasadena, California ATTN: Dr. Georges C. Weill
Electronics Division Rand Corporation 1700 Main Street Santa Monica, California ATTN: Dr. Robert Kalaba	(3) Mr. Robert Brockhurst Woods Hole Oceanographic Institute Woods Hole, Mass.	Naval Research Laboratory Washington 25, D. C. ATTN: Henry J. Passerini, Code 5278A
National Bureau of Standards Washington, D. C. ATTN: Dr. W. K. Saunders	National Bureau of Standards Boulder, Colorado ATTN: Dr. R. Gallet	Dr. George Kear 5 Culver Court Orinda, California
Applied Mathematics and Statistics Lab. Stanford University Stanford, California ATTN: Dr. Albert H. Bowker	Dr. Solomon L. Schwebel 3689 Louisa Road Palo Alto, California	Brooklyn Polytechnic 85 Livingston Street Brooklyn, New York ATTN: Dr. Nathan Marcuvitz
Department of Physics and Astronomy Michigan State College East Lansing, Michigan ATTN: Dr. A. Leitner	University of Minnesota The University of Library Minneapolis 14, Minnesota ATTN: Exchange Division	Department of Electrical Engineering Brooklyn Polytechnic 85 Livingston Street Brooklyn, New York ATTN: Dr. Jerry Shmoys
University of Tennessee Knoxville, Tennessee ATTN: Dr. Fred A. Ficken	Department of Mathematics University of California Berkeley, California ATTN: Professor Bernard Friedman	Department of Mathematics University of New Mexico Albuquerque, New Mexico ATTN: Dr. I. Kolodner
Lebanon Valley College Annville, Pennsylvania ATTN: Professor B.H. Bissinger	Lincoln Laboratory Massachusetts Institute of Technology P. O. Box 73 Lexington 73, Massachusetts ATTN: Dr. Shou Chin Wang, Room C-351	Mathematics Department Polytechnic Institute of Brooklyn Johnson and Jay Street Brooklyn, New York ATTN: Dr. Harry Hochstadt
General Atomic P. O. Box 608 San Diego 12, California Attn: Dr. Edward Gerjuoy	Melpar, Inc., 3000 Arlington Boulevard Falls Church, Virginia ATTN: Mr. K. S. Kelleher, Section Head	Ballistics Research Laboratory Aberdeen Proving Grounds Aberdeen, Maryland ATTN: Dr. Pullen Keats
Department of Physics Amherst College Amherst, Mass. ATTN: Dr. Arnold Arons	Hq. Air Force Cambridge Research Center Laurence G. Hanscom Field Bedford, Mass. ATTN: Mr. Francis J. Zucker, CRRD	Dr. Lester Kraus 4935 Whitehaven Way San Diego, California
California Institute of Technology 1201 E. California Street Pasadena, California ATTN: Dr. A. Erdelyi	Hq. Air Force Cambridge Research Center Laurence G. Hanscom Field Bedford, Mass. ATTN: Dr. Philip Newman, CRRK	University of Minnesota Institute of Technology Minneapolis, Minnesota Attn: Dean Athelston Spilhaus
Mathematics Department Stanford University Stanford, California ATTN: Dr. Harold Levine	Mr. N. C. Gerson Trapelo Road South Lincoln, Mass.	Ohio State University Columbus, Ohio Attn: Prof. C. T. Tai Department of Electrical Eng.
University of Minnesota Minneapolis 14, Minnesota ATTN: Professor Paul C. Rosenbloom	Dr. Richard B. Barrar Systems Development Corp. 2400 Colorado Avenue Santa Monica, California	Naval Research Laboratories Washington 25, D. C. Attn: W. S. Ament, Code 5271
Department of Mathematics Stanford University Stanford, California ATTN: Professor Bernard Epstein	Columbia University Hudson Laboratories P.O. Box 239 145 Palisade Street, Dobbs Ferry, N. Y. ATTN: Dr. N. W. Johnson	Naval Research Laboratory Washington 25, D. C. Attn: Dr. Leslie G. McCracken, Jr. Code 3933A
Applied Physics Laboratory The Johns Hopkins University 8621 Georgia Avenue Silver Spring, Maryland ATTN: Dr. B. S. Gourary	Institute of Fluid Dynamics and Applied Mathematics University of Maryland College Park, Maryland ATTN: Dr. Elliott Montroll	Office of Naval Research Department of the Navy Attn: Geophysics Branch, Code 416 Washington 25, D. C.
(2) Exchange and Gift Division The Library of Congress Washington 25, D. C.	Department of Electrical Engineering Washington University Saint Louis 5, Mo. ATTN: Professor J. Van Bladel	Office of Chief Signal Officer Signal Plans and Operations Division Attn: SIGOL-2, Room 20 Com. Liaison Br., Radio Prop. Sect. The Pentagon, Washington 25, D. C.
Electrical Engineering Department Massachusetts Institute of Technology Cambridge 39, Mass. ATTN: Dr. L. J. Chu	Department of the Navy Office of Naval Research Branch Office 1030 E. Green Street Pasadena 1, California	Defence Research Member Canadian Joint Staff 2001 Connecticut Street Washington, D. C.
Nuclear Development Associates, Inc. 5 New Street White Plains, New York ATTN: Library	Brandeis University Waltham, Mass. ATTN: Library	Central Radio Prop. Lab. National Bureau of Standards Attn: Technical Reports Library Boulder, Colorado
California Institute of Technology Electrical Engineering Pasadena, California ATTN: Dr. Zohrab A. Kaprielian	General Electric Company Microwave Laboratory Electronics Division Stanford Industrial Park Palo Alto, California ATTN: Library	U. S. Weather Bureau U. S. Department of Commerce Washington 25, D. C. Attn: Dr. Harry Wexler

Federal Communications Commission
Washington 25, D. C.
Attn: Mrs. Barbara C. Grimes, Librarian

Upper Atmosphere Research Section
Central Radio Propagation Laboratory
National Bureau of Standards
Boulder, Colorado

Argonne National Laboratory
P.O. Box 299
Lemont, Illinois
Attn: Dr. Hoylande D. Young

Bell Telephone Labs.
Murray Hill, New Jersey
Attn: Dr. S. O. Rice, 3B - 203

Carnegie Institute of Washington
Dept. of Terrestrial Magnetism
5241 Broad Branch Road, N. W.
Washington 15, D. C.
Attn: Library

Georgia Tech Research Institute
225 N. Avenue, N. W.
Attn: Dr. James E. Boyd
Atlanta, Georgia

University of Maryland
College Park, Maryland
Attn: Dr. A. Weinstein
Institute of Fluid Dynamics

Massachusetts Institute of Technology
Lincoln Laboratory
P. O. Box 73
Lexington 73, Massachusetts
Attn: Prof. Radford, Division 3 Head

Willow Run Research Center
University of Michigan
Willow Run Airport
Ypsilanti, Michigan
Attn: Dr. C. L. Dolph

School of Engineering
New York University
University Heights
New York, New York

Shell Fellowship Committee of the
Shell Companies Foundation, Inc.
50 West 50th Street
New York 20, N. Y.
Attn: Mr. J. R. Janssen

Esso Research and Engineering Co.
P. O. Box 51
Linden, New Jersey
Attn: Mr. C. L. Brown, Manager

Union Carbide and Carbon Corp.
30 E. 42nd Street
New York 17, New York
Attn: Mr. L. E. Erlanson

Convair
San Diego 12, California
Attn: Mr. Marvin Stern

Bell Telephone Labs., Inc.
463 West Street
New York 13, N. Y.
Attn: Dr. Mervin J. Kelly

Engineering Library
University of California
405 Hilgard Avenue
Los Angeles 24, California

Convair, A Division of General Dynamics Corp.
Daingerfield, Texas
Attn: J. E. Arnold, Division Manager

Convair, A Division of General Dynamics Corp.
San Diego 12, California
Attn: R. L. Bayless, Chief Engineer

Convair, a Division of General Dynamics Corp.
San Diego 12, California
Attn: K. J. Bossart, Chief Engineer-WS107A

Convair, A Div. of General Dynamics
Corp.
Fort Worth 1, Texas
Attn: F. W. Davis, Chief Engineer

Convair, A Div. of General Dynamics
Corp.
Pomona, California
Attn: C. D. Perrine
Asst Div. Manager, Engin.

Shell Development Company
Exploration and Production Res. Div.
3737 Bellaire Boulevard
Houston 25, Texas
Attn: Miss Aphrodite Mamoulides

RCA Laboratories
Princeton, New Jersey
Attn: Dr. Charles Polk

Stanford Research Institute
S. Pasadena, California
Attn: Dr. J. Brandstatten

Wayne State University
Kresge-Hooker Science Library
5250 Second Boulevard
Detroit 2, Michigan

ARRA
1 Bond Street
Westbury, L. I., New York
Attn: Dr. Norman Spector

Varian Associates
611 Hansen Way
Palo Alto, California
Attn: Mrs. Perry Conway
Technical Librarian

Case Institute of Technology
Department of Electrical Engin.
University Circle
Cleveland 6, Ohio
Attn: Prof. Robert Plonsey

Dr. Ming S. Wong, CRRKP
Air Force Cambridge Research Center
Laurence G. Hanscom Field
Bedford, Massachusetts

Physics Section
AVCO-RAD Division
20 South Union Street
Lawrence, Mass.
Attn: Dr. Ernest Bauer

Advanced Development Section
Western Development Labs.
Philco Corp.
3875 Fabian Way
Palo Alto, California
Attn: Dr. Albert R. Giddis

Department of Aeronautical Engineering
University of Michigan
Ann Arbor, Michigan
Attn: Prof. Mahinder Uberoi

Gordon McKay Laboratory
Harvard University
Cambridge 39, Mass.
Attn: Dr. S. R. Seshadri

Commander
Air Research and Development Command
Attn: RDTR
Andrews Air Force Base
Washington 25, D. C.

College of Engineering
Dept. of Electrical Engineering
University of Florida
Gainesville, Florida
Attn: Dr. Sullivan

Dr. V. M. Papadopoulos
Dept. of Engineering
Brown University
Providence, R. I.

Major Vernon Lee Dawson
RSDD-OML(MO)
Redstone Arsenal
Huntsville, Alabama

Grumman Aircraft Engineering Corp.
South Oyster Bay Road
Bethpage, Long Island, N. Y.
Attn: Dr. Charles Mack

AF Office of Scientific Research
Washington 25, D. C.
Attn: Dr. Karl Kaplan

University of California
Radiation Laboratory
P. O. Box 808
Livermore, California
Attn: Dr. Bernard A. Lippmann

Department of Electrical Engineering
Case Institute of Technology
University Circle
Cleveland 6, Ohio
Attn: Professor Albert E. Collin

Antenna Laboratory
Air Force Cambridge Research Center
Laurence G. Hanscom Field
Bedford, Massachusetts
Attn: Mr. Philip Blackstone

Lt. Mark J. Beran, CRRD
Air Force Cambridge Research Center
Laurence G. Hanscom Field
Bedford, Massachusetts

Mr. Richard Mack, CRRD
Air Force Cambridge Research Center
Laurence G. Hanscom Field
Bedford, Massachusetts

System Development Corporation
2500 Colorado Avenue
Santa Monica, California
Attn: Library

[illegible]

**N. Y. U. Institute of
Mathematical Sciences**
25 Waverly Place
New York 3, N. Y.

